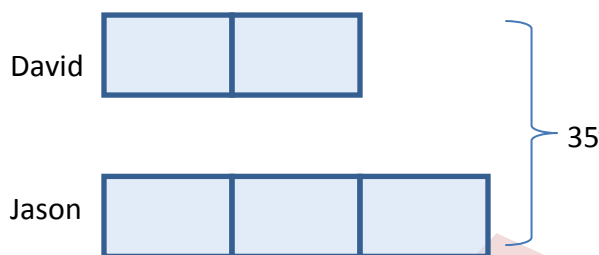


# Introduction to Tape Diagrams

**Tape diagrams are best used to model ratios when the two quantities have the same units.**

## Basic Ratios

**1. David and Jason have marbles in a ratio of 2:3. Together, they have a total of 35 marbles. How many marbles does each boy have?**



Tape diagrams are visual models that use rectangles to represent the parts of a ratio. Since they are a visual model, drawing them requires attention to detail in the setup. In this problem David and Jason have numbers of marbles in a ratio of 2:3. This ratio is modeled here by drawing 2 rectangles to represent David's portion, and 3 rectangles to represent Jason's portion. The rectangles are uniform in size and lined up, e.g., on the left hand side, for easy visual reference. The large bracket on the right denotes the total number of marbles David and Jason have (35). It is clear visually that the boys have 5 rectangles worth of marbles and that the total number of marbles is 35. This information will be used to solve the problem.

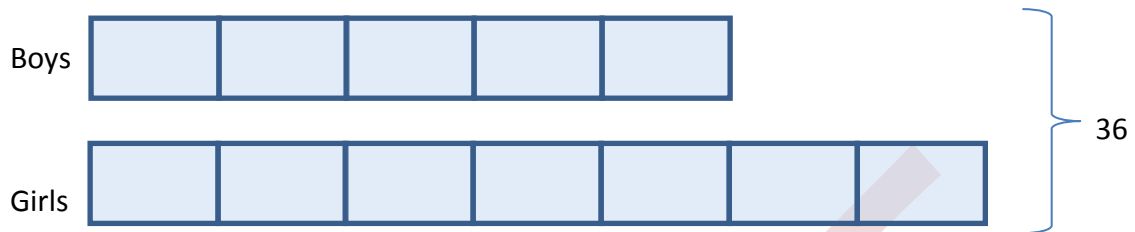
5 rectangles = 35 marbles      Dividing both numbers of rectangles and marbles by 5

1 rectangle = 7 marbles      This information will be used to solve the problem.

David has 2 rectangles and  $2 \times 7 = 14$  marbles. Therefore David has 14 marbles.

Jason has 3 rectangles and  $3 \times 7 = 21$  marbles. Therefore Jason has 21 marbles.

**2. The ratio of boys to girls in the class is 5:7. There are 36 children in the class. How many more girls than boys are there in the class?**



This problem is set up like the previous one. The reasoning will also begin in the same way.

12 rectangles = 36 children    By dividing both numbers of rectangles and children by 12

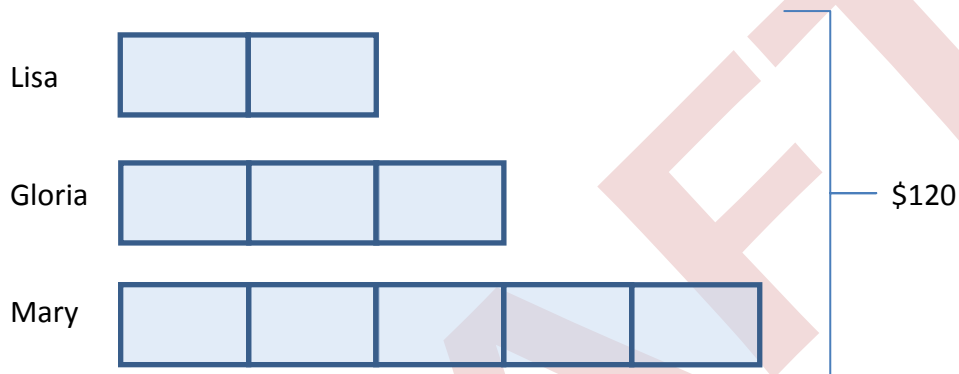
1 rectangle = 3 children    Using the visual model above and the rectangle comparison, this problem could be solved the same way as the one above. This would be done by figuring out the number of boys and the number of girls and then subtracting. However, using the visual model, there is a second approach. When looking at the model, it is clear that the girls have two more rectangles or parts than the boys. Since 1 rectangle = 3 children then 2 rectangles = 6 children.

There are 6 more girls than boys in the class.

### Comparing Ratios of 3 Items

Tape diagrams allow students to approach more complex problems that involve comparing ratios of 3 items. This would be more difficult to solve without the use of a tape diagram, but with a tape diagram the structure of the problems can be visualized and, sometimes, the solutions are readily available.

3. Lisa, Megan and Mary were paid \$120 for babysitting in a ratio of 2: 3: 5. How much less did Lisa make than Mary?



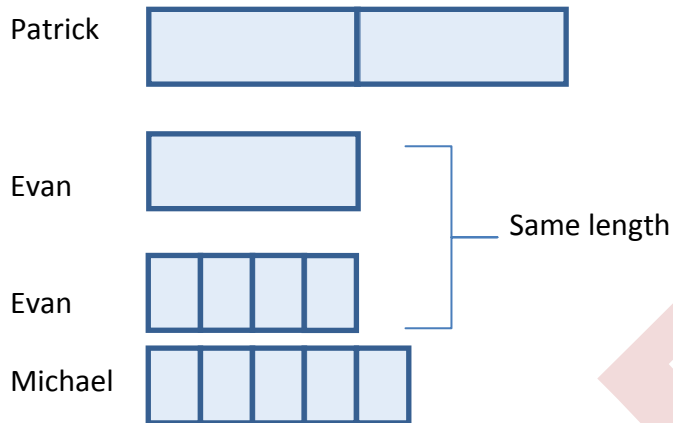
10 rectangles = \$120

1 rectangle = \$12

Since Lisa made 3 rectangles less than Mary, she made \$36 less than Mary.

## Comparing 2 Different Ratios

4. The ratio of Patrick's M & M's to Evan's is 2: 1 and the ratio of Evan's M & M's to Michael's is 4: 5. Find the ratio of Patrick's M & M's to Michael's.

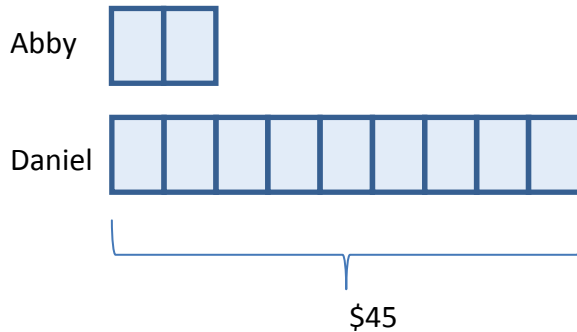


This problem begins with comparing Patrick to Evan. Patrick gets two rectangles (parts) to Evan's one. Next Evan is compared to Michael. The Length of Evan's rectangle cannot change, but this time it is broken into 4 smaller rectangles. Michael's five rectangles are drawn the same size as Evans. Now it is possible to break Patrick's rectangles into pieces that can be compared to Michael's. Since Evan's original rectangle is equal to four of the smaller rectangles, then it follows that each of Patrick's rectangles is equal to 4 of the smaller rectangles. Using that comparison, Patrick has 2 large rectangles, which can be each broken into 4 smaller rectangles, giving him a total of 8 smaller rectangles. Since the smaller rectangles are the same size as Michael's, they can be compared.

The ratio of Patrick's M & M's to Michael's is 8 : 5.

## Changing Ratios

5. The ratio of Abby's money to Daniel's is 2: 9. Daniel has \$45. If Daniel gives Abby \$15, what will be the new ratio of Abby's money to Daniel's?

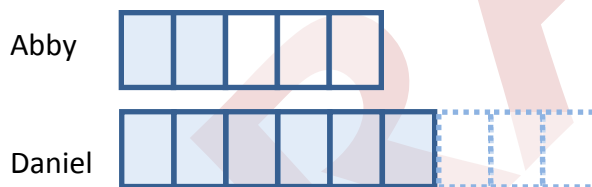


As in previous problems, the problem starts with the same setup. Since the ratio of Abby's money to Daniel's is 2: 9, Abby is given 2 rectangles, while Daniel is given 9. The problem states that Daniel has \$45, which is broken into 9 rectangles.

$$9 \text{ rectangles} = \$45$$

$$1 \text{ rectangle} = \$5$$

Daniel gives Abby \$15, which is equal to 3 rectangles. Now the comparison looks like this.



Based on the new model, it is possible to determine the current ratio of Abby's money to Daniel's money. The ratio of Abby's money to Daniel's is 5 : 6.

It would be possible to extend the question further. Students could determine how much Abby had at the beginning, and how much they each have now. The tape diagram lends itself to illustrating/modeling many different types of problems.