

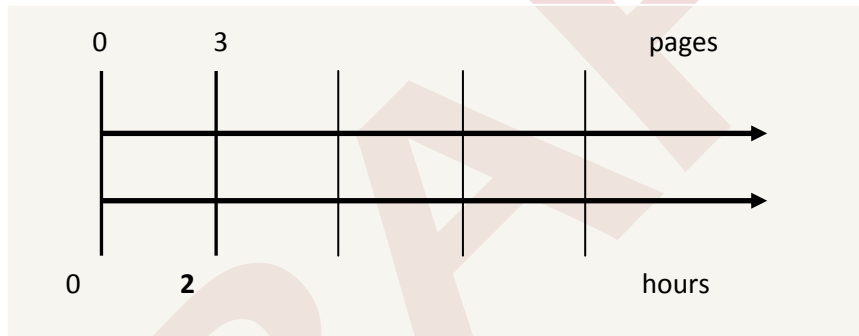
Introduction to Double Number Line Diagrams

Double number line diagrams are best used when the quantities have different units. Double number line diagrams can help make visible that there are many, even infinitely many, pairs of numbers in the same ratio—including those with rational number entries. As in tables, unit rates (R) appear in the pair $(R, 1)$.

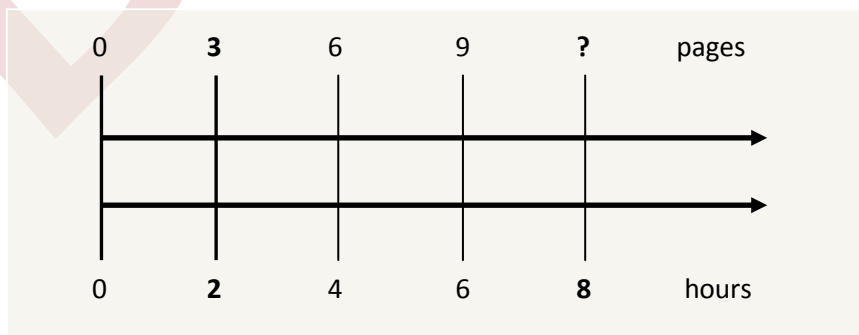
Finding average rate

It took Megan 2 hours to complete 3 pages of math homework. Assuming she works at a constant rate, if she works for 8 hours, how many pages of math homework will she complete? What is the average rate at which she works?

Solution:

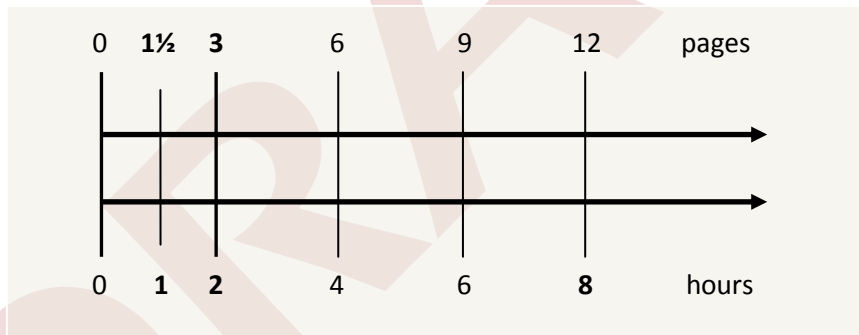


To solve this problem it makes sense to use a double number line rather than a tape diagram; the double number line lends itself to the comparison of quantities in two different units, e.g., pages and hours. To complete a double number line like the one above, first label the pages on the top line. Starting with zero, mark the top line with multiples of 3, since Megan completes 3 pages in every time interval. On the bottom line label hours, this time going by multiples of 2 since the time interval is 2 hours.



It is now possible to answer the first part of the question “*if she works for 8 hours, how many pages of math will she complete?*” Since the number of hours is given and the number of pages is the unknown, it is necessary to solve the problem by looking at the bottom number line, which is in hours. On the bottom number line, find and label the time of 8 hours. The corresponding number of pages is on the upper number line and connected to the 8 hour mark; in this case it is 12. So, at this rate, Megan will complete 12 pages in 8 hours.

The second part of the question requires a slightly different approach to extract information from the double number line. The question “*what is the average rate at which she works?*” is asking for a unit rate. Here is an example of unit rate: if one travels 240 miles in 4 hours, then the unit rate is 60 miles **per** hour or 60 miles **for each** hour. Unit rates involve the expression: “**Per something**” which means “**for each something**”. In this particular problem the unit rate is in the form of number of pages per hour or number of pages for each hour. This means the time interval associated with the unit rate is 1 hour. Since 1 hour is not already on the double number line, it needs to be located and added to the bottom number line. Therefore the space between 0 and 2 on the bottom number line is divided in half and the label of 1 (for 1 hour) is added.



Since 1 hour is halfway between 0 and 2 on the hour number line, the corresponding number of pages will be half way between 0 and 3 on the page number line. Halfway from 0 to 3 is $1\frac{1}{2}$. This is labeled on the page number line. It is now possible to answer the question “*What is the average rate at which she works?*”. The answer is $1\frac{1}{2}$ pages per hour.