MATH NEWS

Grade 8, Module 4, Topic A

LAFAYETTE PARISH SCHOOL SYSTEM

8th Grade Math

Module 4: Linear Equations

Math Parent Letter

This document is created to give parents and students a better understanding of the math concepts found in Eureka Math (© 2013 Common Core, Inc.) that is also posted as the Engage New York material which is taught in the classroom. Module 4 of Eureka Math (Engage New York) builds on what students already know about unit rates and proportional relationships to linear equations and their graphs. Students understand the connections between proportional relationships, lines, and linear equations in this module. Also in this module, students learn to apply the skills they acquired in Grades 6 and 7, with respect to symbolic notation and properties of equality to transcribe and solve equations in one variable and then in two variables.

Focus Area Topic A:

Writing and Solving Linear Equations

Students will begin translating written statements into mathematical expressions or equations using symbols. Then, students write linear and non-linear expressions leading to linear equations, which are solved using properties of equality. Students learn that not every linear equation has a solution. In doing so, students learn how to rewrite given equations into simpler forms until an equivalent equation results in a unique solution, no solution, or infinitely many solutions Throughout Topic A students must write and solve linear equations in real-world and mathematical situations.

Words to Know:

Linear Expressions- sums of constants and products of constants and x raised to a power of **0** or **1**.
Non-linear expression – sums of constants and products of constants and an x raised to a power that is **not 0** or **1.**
Equation - a statement of equality between two expressions.
Term - any product of an integer power of x and a constant or just a constant.
Constants - fixed numbers
Coefficient - When a term is the product of a constant(s) and a power of x, the constant is called a **coefficient**.

Writing Equations Using Symbols

Writing in symbols is simpler than writing in words, as long as everyone involved is clear about what the symbols mean. When we write mathematical statements using letters, we say we are using symbolic language.

Example 1: Paulo has a certain amount of money. If he spends six dollars, then he has $\frac{1}{4}$ of the original amount left. Use x to represent the amount of money Paulo had originally.

What he spent

Paulo's \$ → $x - 6 = \frac{1}{4}x$ ← Remaining amount

Focus Area Topic A:

Writing and Solving Linear Equations

Writing Equations Using Symbols (continued)

Example 2: Represent the statement "The sum of three consecutive integers is 372."

(Consecutive means "one after the other")
x ⟶ 1st integer
x + 1⟶ 2nd integer
x + 2⟶ 3rd integer and so

So: $x + x + 1 + x + 2 = 372$ *or* $3x + 3 = 372$

Linear and Non-Linear Expressions in x

A linear expression in x is an expression where each term is either a constant, an x, or a product of a constant and x.

$3 + x + 7x$

More precisely, a linear equation is one that is dependent only on constants and a variable raised to the first power.

$5x+3$ ***is*** a linear expression

$2x^2 + 9x + 5$ ***is not*** a linear expression (*The term $2x^2$ does not fit the definition of a linear expression in x because the variable is raised to a power greater than 1.)*

The reason we want to be able to distinguish linear expressions from non-linear expressions is because we will soon be solving linear equations.

Linear Equations in x

A linear equation in x is actually a question. "Can you find all numbers x, if they exist, that satisfy a given equation?"

We begin by considering linear equations in x whose solution will be a number. When all instances of x are replaced with the number, the left side will equal the right side.

Here is a linear equation in x: $8x - 19 = -4 - 7x$.
Is 5 a solution to the equation? That is, is $x = 5$ a solution to the equation?

Left side	*Right side*
$8x - 19$	$-4 - 7x$
$8 \cdot 5 - 19$	$-4 - 7 \cdot 5$
$40 - 19$	$-4 - 35$
21	-39

No, because the left side does not equal the right side; therefore $x \neq 5$.

Focus Area Topic A:
Writing and Solving Linear Equations

Solving a Linear Equation

To solve an equation means to find all of the numbers x, if they exist, so that the given equation is true.

Step 1: Subtract 13 from both sides of the equal sign.

Step 2: Add 9x to both sides to put all terms with x on the same side of the equal sign.

Step 3: Multiply both sides by $\frac{5}{51}$ to get a solution in the form of x.

$$\frac{6}{5}x + 13 = 23 - 9x$$
$$\frac{6}{5}x + 13 - \mathbf{13} = 23 - \mathbf{13} - 9x$$
$$\frac{6}{5}x = 10 - 9x$$
$$\frac{6}{5}x + \mathbf{9x} = 10 - 9x + \mathbf{9x}$$
$$\frac{51}{5}x = 10$$
$$\frac{51}{5}x \bullet \frac{5}{51} = \frac{10}{1} \bullet \frac{5}{51}$$
$$x = \frac{50}{51}$$

$$\frac{51}{5} \bullet \frac{5}{51} = 1$$

Writing and Solving Linear Equations

Example:

One angle is five less than three times the size of another angle. Together they have a sum of 143°. What is the size of each angle? Let x represent the size of first angle.

Size of first angle → x; Size of second angle → $3x - 5$

$$x + 3x - 5 = 143$$
$$4x - 5 = 143$$
$$4x - 5 + \mathbf{5} = 143 + \mathbf{5}$$
$$4x = 148$$
$$x = 37$$

The first angle is 37°. The second angle is $3(37) - 5$ which is 106°. (To verify the solution, check that left and right sides are equal.)

Solutions of a Linear Equation

Example: What value of x would make the linear equation $4x + 3(4x + 7) = 4(7x + 3) - 3$ true?

Step 1: Use distributive property on both sides to deal with the parenthesis.
Step 2: Combine like terms.
Step 3: Subtract 16x from both sides to get x terms together.
Step 4: Subtract 9 from both sides
Step 5: Divide both sides by 12.

$$4x + 3(4x + 7) = 4(7x + 3) - 3$$
$$4x + 12x + 21 = 28x + 12 - 3$$
$$16x + 21 = 28x + 9$$
$$16x - \mathbf{16x} + 21 = 28x - \mathbf{16x} + 9$$
$$21 = 12x + 9$$
$$21 - \mathbf{9} = 12x + 9 - \mathbf{9}$$
$$12 = 12x$$
$$1 = x$$
$$x = 1$$

Students will discover not every linear equation has a solution. Consider the following equation:

$2(x+1)=2x-3$. What value of x makes the equation true?

$$2(x + 1) = 2x - 3$$
$$2x + 2 = 2x - 3$$
$$2x - \mathbf{2x} + 2 = 2x - \mathbf{2x} - 3$$
$$2 \neq 3$$

Since $2 \neq 3$, then the equation has no solution.

Classification of Solutions

There are three classifications of solutions to linear equations: one solution (unique solution), no solution, or infinitely many solutions.

Equation with EXACTLY ONE SOLUTION

$$4x + 3 = 3x + 5$$

constants are different; coefficients of x are different

Equation with NO SOLUTION

$$4x + 3 = 4x + 5$$

constants are different; coefficients of x are the same

Equation with INFINITE SOLUTIONS

$$4x + 8 = 4x + 8$$

constants are the same; coefficients of x are the same

Linear Equations in Disguise

Some equations that may not look like linear equations are, in fact, linear. Proportions are linear equations in disguise and are solved the same way we normally solve proportions.

Tip: To get rid of fractions, multiply by a common denominator. In this case a product of the two denominators.

Example: $\frac{7}{3x+9} = \frac{1}{8}$

$$8 \cdot (3x + 9) \cdot \frac{7}{3x+9} = \frac{1}{8} \cdot 8 \cdot (3x + 9)$$
$$7(8) = (3x + 9)1$$
$$56 = 3x + 9$$
$$56 - \mathbf{9} = 3x + 9 - \mathbf{9}$$
$$47 = 3x$$
$$\frac{47}{3} = x$$

The equation $\frac{47}{3} = x$ is clearly a linear equation.

Applications of Linear Equations

Students apply the skills learned to this point, namely writing and solving linear equations in one-variable, to model real-world problems.

Example:

A cell phone company charges \$30 per month and \$0.07 per minute. If Robert's January cell phone bill is \$59.40, how many minutes did he talk?

- Identify a variable: Let x represent the number of minutes Robert talked.
- Write equation: $30 + 0.07x = 59.40$
- Subtract 30 from both sides: $30 - \mathbf{30} + 0.07x = 59.40 - \mathbf{30}$
- Divide both sides by 0.07: $\frac{0.07x}{\mathbf{0.07}} = \frac{29.40}{\mathbf{0.07}}$

Robert talked 420 minutes. $\mathbf{x = 420}$