8th Grade Math

Module 4: Linear Equations

Math Parent Letter

This document is created to give parents and students a better understanding of the math concepts found in Eureka Math (© 2013 Common Core, Inc.) that is also posted as the Engage New York material which is taught in the classroom. Module 4 of Eureka Math (Engage New York) builds on ratios, rates, and unit rates to formally define proportional relationships and the constant of proportionality.

Focus Area Topics D & E: *Linear Equations*

Words to Know

System of Linear Equations A system of linear equations, also referred to as simultaneous linear equations, is the set of at least two linear equations.

Example: $\begin{cases} x + y = 15 \\ 2x - y = 12 \end{cases}$ is a system of linear equations.

Solution to a system of linear equations If an equation has two variables, then a solution is a part of numbers from the domain of the variables that, when each number from the pair is substituted into all instances of its corresponding variable, makes the equation a true number sentence. In other words, a solution to a system of equations is an ordered pair (x, y) that satisfies both equations at the same time.

Example: The solution to the system of linear equations is $\begin{cases} x + y = 15 \\ 2x - y = 12 \end{cases}$ is the ordered pair $(9, 6)$ because the ordered pair is a solution to each linear equation of the system **and** it is the point on the plane where the graphs of the two equations intersect.

Focus Area D

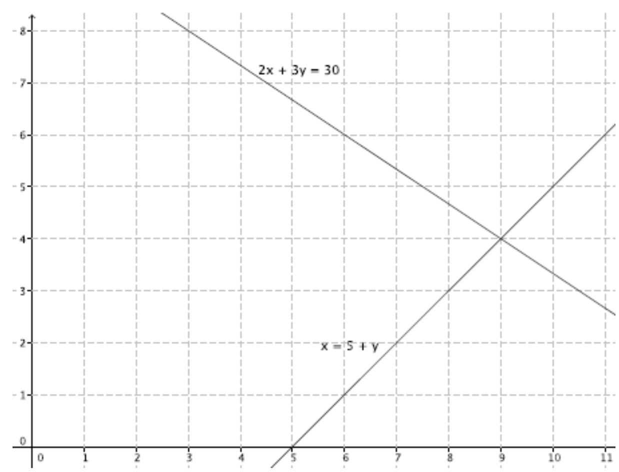
Introduction to System of Equations

This topic focuses on three methods for solving a system of linear equations which are graphing, substitution and eliminations. Students will learn when two equations are involved in the same problem and work must be completed on them simultaneously.

Real-life example: Derek scored 30 points in the basketball game he played and not once did he go to the free throw line. This means Derek scored two point shots and three point shots. Derek tells you that the number of two-point shots that he made is five more than the number of three-point shots. Write two equations that represent Derek's game. Let *x* represent the number of two-pointers and *y* represent the number of three-pointers. Students will be asked how many two point shots and three point shots did Derek make in his game.

$$\begin{cases} 2x + 3y = 30 \\ x = 5 + y \end{cases}$$

Ultimately, our goal is to determine the exact location on the coordinate plane where the graphs of the two linear equations intersect, giving us the ordered pair (x, y) that is the solution to the system of equations.



The point of intersection of the two lines is (9, 4). Therefore, Derek made 9 two-point shots and 4 three-point shots.

Solving Systems of Equations by Graphing

Students will apply their knowledge of graphing linear equations using slope and the y-intercept as well as using the x and y intercepts. Students will graph the system of equations to find the point of intersection which is the solution since this ordered pair that both equations have in common. Students will learn to verify that the point of intersection is a solution to each equation of the simultaneous equations.

Example:

Graph the linear system on a coordinate plane:

$$\begin{cases} 2y + x = 12 \\ y = \dfrac{5}{6}x - 2 \end{cases}$$

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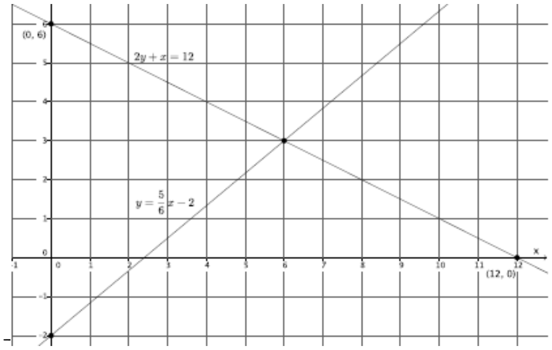

a. Name the ordered pair where the graph of the two linear equations intersect. **(6, 3)**

b. Verify that the ordered pair named in (a) is a solution to the first equation, 2y + x = 12.

$$\mathbf{2(3) + 6 = 12}$$
$$\mathbf{6 + 6 = 12}$$
$$\mathbf{12 = 12}$$

The left and right sides of the equation are equal.

c. Verify that the ordered pair named in (a) is a solution to the second equation, $y = \frac{5}{6}x - 2$.

$$\mathbf{3 = \frac{5}{6}(6) - 2}$$
$$\mathbf{3 = 5 - 2}$$
$$\mathbf{3 = 3}$$

The left and right sides of the equation are equal.

Characterization of Parallel Lines

Students learn that a system can consist of parallel lines and thus produce an answer of "no solution," because the lines will not intersect.

$$\text{Graph the system} \begin{cases} y = \dfrac{2}{3}x + 4 \\ y = \dfrac{4}{6}x - 3 \end{cases}$$

The slopes of the equations are $\frac{2}{3}$ and $\frac{4}{6}$ respectively. However, $\frac{4}{6} = \frac{2}{3}$ when reduced to lowest terms. The slopes of the two equations are equal. Parallel lines have the same slope and by definition will not intersect.

Nature of Solutions of a System of Linear Equations

Students will examine systems that have infinitely many solutions which mean the quantity of solutions is unlimited.

Example $\begin{cases} 6x + 4y = 10 \\ 3x + 2y = 5 \end{cases}$

The second equation is a multiple of the first. For instance, multiply the second equation by 2.

$$3x + 2y = 5$$
$$2(3x + 2y = 5)$$
$$2(3x) + 2(2y) = 2(5)$$
$$6x + 4y = 10$$

Your answer produces the first equation. Therefore, this system of equations has infinite many solutions.

Solving Systems of Linear Equations by Substitution Method

Students learn how to solve a system of equations using the substitution method when one of the equations has originally been solved for x or y. Consider the system of equations: y = x – 3 and 3x + y = 17. The substitution method will require the student to substitute "x – 3" in the place of "y" in the second equation and then solve for x.

$$3x + y = 17$$
$$3x + (x - 3) = 17$$
$$4x - 3 = 17$$
$$4x = 20$$
$$\frac{4x}{4} = \frac{20}{4}$$
$$x = 5$$

Substitute 5 for the value of x in either equation.

$$y = x - 3$$
$$y = 5 - 3$$
$$y = 2$$

The solution is (5, 2).

Solving Systems of Linearn Equations by Elimination Method

Students learn how to solve a system of equations using elimination especially when the equations are in standard form, $ax + by = c$. The purpose is simply to eliminate one of the variables making the equation a single variable equation. We accomplish this by adding the two equations when two terms are opposites (positive/negative) or subtracting the two equations when two terms are the same. When no pair of terms are the same or opposites, we can eliminate a variable by multiplying every term in a single equation by the same constant in order to create the same or opposite terms. Sometimes you may need to multiply both equations by different constants to create the same or opposite terms.

Consider $-3x + 6y = 21$ and $3x + 2y = -5$. Since $-3x$ and $3x$ are opposites, students will add the equations and then the x terms will be eliminated.

$$-3x + 6y + (3x + 2y) = 21 + (-5)$$
$$-3x + 3x + 6y + 2y = 16 \quad \text{(x terms had a sum of zero)}$$
$$8y = 16$$
$$y = 2$$

Next, the student will substitute 2 in the place of y in either equation. The resulting equation will be solved for x to find that x = -3. The solution is (-3, 2).

Focus Area E

Pythagorean Theorem

In lesson 31 students will learn how to apply what they learned about systems of linear equations to find a Pythagorean triple using the Babylonian method. Basically, students will learn to use a system of equations to find three numbers, a, b, and c, so that $a^2 + b^2 = c^2$.