MATH NEWS

Grade 8, Module 2, Topic C

8^{th} Grade Math

Module 2: The Concept of Congruence

Math Parent Letter

This document is created to give parents and students a better understanding of the math concepts found in Eureka Math (© 2013 Common Core, Inc.) that is also posted as the Engage New York material which is taught in the classroom. Module 2 of Eureka Math (Engage New York) focuses on translations, reflections, and rotations in the plane and precisely defines the concept of *congruence.*

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Focus Area Topic C:

Definitions and Properties of Basic Rigid Motions

Words to Know:

Transformation – a rule, to be denoted by *F*, that assigns each point *P* of the plane a unique point which is denoted by *F*(*P*).

Basic Rigid Motion – a basic rigid motion is a rotation, reflection, or translation of the plane.

Translation – a basic rigid motion that moves a figure along a given vector.

Rotation – a basic rigid motion that moves a figure around a point, *d* degrees.

Reflection – a basic rigid motion that moves a figure across a line.

Sequence (Composition) of Transformations – more than one transformation – given transformations *G* and *F*, $G \circ F$ is called the composition of *F* and *G*.

Vector – a Euclidean vector (or directed segment) $\overrightarrow{AB}$ is the line segment AB together with a direction given by connecting an initial point *A* to a terminal point *B*.

Congruence – a sequence of basic rigid motions (rotations, reflections, translations of the plane: symbol for congruence: ≅

Transversal – given a pair of lines *L* and *M* in a plane, a third line *T* is a transversal if it intersects *L* at a single point and intersects *M* at a single but different point.

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Definition of Congruence and Some Basic Properties

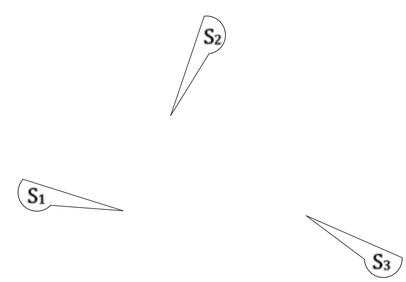
The concept of congruence is defined as "mapping one figure onto another using a sequence of basic rigid motions." Students learn that to prove two figures are congruent there must be a sequence of rigid motions that maps one figure onto the other.

Focus Area Topic C:

Congruence and Angle Relationships

The following in an example from **Lesson 11's Class Exercises.**

1. Describe the sequence of basic rigid motions that shows that $S_1 \cong S_2$ (S_1 is congruent to S_2).
2. Describe the sequence of basic rigid motions that shows that $S_2 \cong S_3$. (S_2 is congruent to S_3)
3. Describe the sequence of basic rigid motions that shows that $S_1 \cong S_3$ (S_1 is congruent to S_3).



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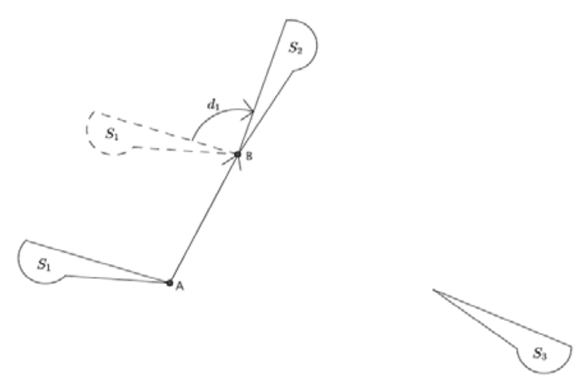
SOLUTION:

1. **Translate along vector AB. Rotate *d* degrees around point B. Reflect across the longest side of the figure so that S_1 maps onto S_2.**



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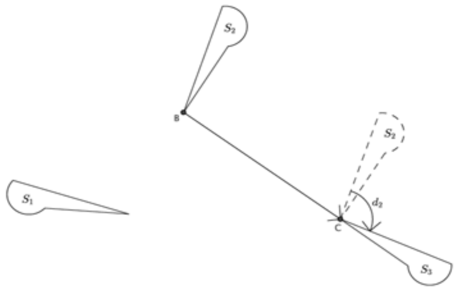
(continued)

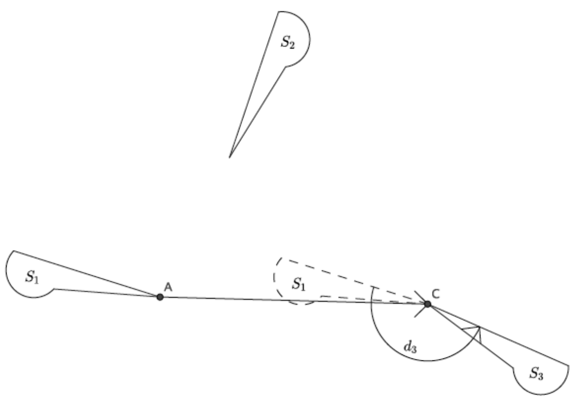
SOLUTION:

2. **Translate along vector BC. Rotate d_2 degrees around point C so that S_2 maps onto S_3.**



SOLUTION:

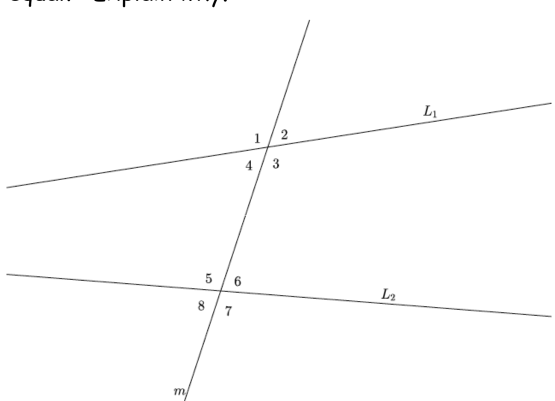
3. **Translate along vector AC. Rotate d_3 degrees around point C. Reflect along the longest side of the figure so that S_1 maps onto S_3.**



Angles Associated with Parallel Lines

EXPLORATORY CHALLENGE

In the figure below, L_1 is not parallel to L_2, and *m* is a transversal. Use a protractor to measure angles 1-8. Which, if any, are equal? Explain why.



SOLUTION: $\angle 1 = \angle 3$, $\angle 2 = \angle 4$, $\angle 5 = \angle 7$, $\angle 6 = \angle 8$

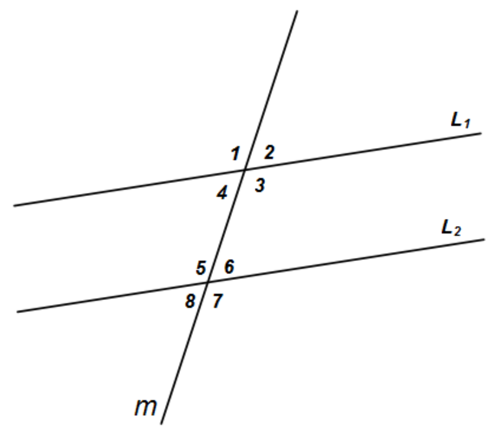
These angle pairs are equal because they are vertical angles. Because you can rotate an angle 180 degrees about its vertex to create vertical angles **and** rotations are degree preserving, vertical angles are always congruent.

A pair of lines cut by a transversal creates many more angle relationships. Angles that are on the same side of the transversal in corresponding positions (above each of L_1 and L_2 or below each of L_1 and L_2) are called **corresponding angles**. (Examples: $\angle 1$ *and* $\angle 5$, $\angle 4$ *and* $\angle 8$, $\angle 2$ *and* $\angle 6$, $\angle 3$ *and* $\angle 7$)

When angles are on opposite sides of the transversal and between (inside) the lines L_1 and L_2, they are called **alternate interior angles**. (Examples: $\angle 4$ *and* $\angle 6$, $\angle 3$ *and* $\angle 5$)

When angles are on opposite sides of the transversal and outside of the parallel lines (above L_1 and L_2), they are called **alternate exterior angles**. (Examples: $\angle 1$ *and* $\angle 7$, $\angle 2$ *and* $\angle 8$)

The next step is to explore angle relationships if L_1 and L_2 are parallel. If L_1 and L_2 are parallel, then a pair of corresponding angles are congruent to each other, alternate interior angle pairs are congruent, and alternate exterior angle pairs are congruent. These relationships are proven using basic rigid motions.

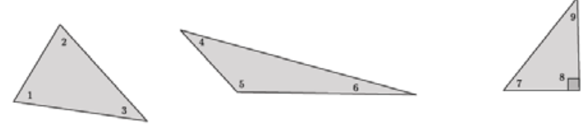


One basic rigid motion would be to translate L_1 down along the transversal until it meets L_2. This would show that $\angle 1$ *and* $\angle 5$ coincide. Since translations preserve angle measures then this shows that $\angle 1$ is congruent to $\angle 5$. Can you think of other basic rigid motions or sequences of basic rigid motions that could also show the congruent angle relationships created by a pair of parallel lines cut by a transversal?

Angle Sum of a Triangle & More on the Angles

The knowledge of rigid motions and angle relationships is utilized to develop informal arguments to show that the sum of the degrees of interior angles of any triangle is 180 degrees.

Concept Development



$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 5 + \angle 6 = \angle 7 + \angle 8 + \angle 9 = 180$

The above is true because: $\angle 1 + \angle 2 + \angle 3 = 180$; $\angle 4 + \angle 5 + \angle 6 = 180$ and $\angle 7 + \angle 8 + \angle 9 = 180$.